THE
Mathematics Student

A Quarterly Dedicated to the service of Students and Teachers of Mathematics in India.

VOL. III MARCH 1935 NO. 1

OFFICE OF THE REGISTRAR OF BOOKS
2-AUG1935
MADRAS

Edited by
A. NARASINGA RAO, M.A., L.T.

With the co-operation of

S. CHOWLA, M.A., Ph.D.;
A. A. KRISHNASWAMI AYYANGAR, M.A.;
S. PURUSHOTHAM, M.A.;
B. RAMAMURTI, M.A.;
V. RAMASWAMI AIYAR, M.A.;
S. SIVASANKARANARAYANA PILLAI, M.Sc.
G. A. SRINIVASAN, M.A., L.T.;
C. N. SRINIVASIENGAR, D.Sc.;
N. SUNDARAM IYER, M.A.
and others.

1125

Published by the Indian Mathematical Society

—

...SEPH'S INDUSTRIAL SCHOOL PRESS, TRICHY
1935

JL
Bm 211, N33 MS
N35
190018

SPHINX

Monthly Review

OF

RECREATIVE QUESTIONS

EDITED BY M. KRAITCHIK

Laureatus of the Institute of France.

PUBLISHED TWELVE TIMES A YEAR.

Subscription : 7 Belgas a Year.

Apply to

SPHINX,
75, Rue Philippe Baucq, Brussels,
BELGIUM.

SCRIPTA MATHEMATICA LIBRARY

NECESSARY ADDITIONS TO YOUR LIBRARY

1. Poetry of Mathematics and Other Essays, by Professor **David Eugene Smith**. 96 pages. Price, in a beautiful blue cloth binding, 75 cents.

2. Mathematics and the Question of Cosmic Mind, with Other Essays, by Professor **Cassius Jackson Keyser**. 128 pages. Price. in beautiful blue cloth binding, 75 cents.

3. Mind, the Maker: The World Theory of the late William Benjàmin Smith. Presented by Professor **Cassius Jackson Keyser.** 32 pages. Price 35 cents.

Address:—

SCRIPTA MATHEMATICA
Amsterdam Avenue and 186th Street,
NEW YORK, N. Y.

THE MATHEMATICS STUDENT

Volume III] MARCH 1935 [Number 1

THE NUMBER CONCEPT

BY

REV. J. B. FREEMAN, *St. Joseph's College, Bangalore.*

A few days ago the writer came across a newspaper report of a speech made by a gentleman at a school in England. He says: "One and one make two"; and to the question he puts himself: "Who made them so?" he gives the answer: "The mind."

We propose to consider the question in detail to a certain extent. In the very simple query are involved difficulties of a physical, of a psychological, and of a metaphysical nature; and the answer is not so simple as that given. Even if it were so verbally, it would imply a great deal.

It is very remarkable, but none the less true, that we eat and digest before we know the how and the why of these operations. We walk before we know anything of the nature of balance; we talk before we know grammar or idiom; we think and we reason before we know aught of the science of logic or of psychology; and we "do" mathematics before we understand its nature. This is true of the individual as well as of the human race. Our lines of thought, which proceed in different directions, group themselves naturally into the great branches of human interest and investigation. It is the aim of all those investigations, called the Sciences, to classify objects and operations, the things about us and the phenomena connected with them; to generalise and to explain the multifarious phenomena by as few "Laws" as possible: in short, to reduce heterogeneity into homogeneity, diversity into unity. While these investigations progress and when they have gone far enough, we enquire into the validity of our assumption, of our mode of procedure and of our conclusions, to see how far we have gone in the direction of truth. We shall leave aside for the present the fundamental queries: What is Truth? Where is it? How can we find

it? It is clear, however, that if our studies proceed by analysis or by synthesis, or both, by induction or by deduction, or both, we shall always reach certain assumptions; and, should we question their legitimacy, we fall into a quagmire, as it were, in which we are stranded; for such a question would imply a doubt as to our right to question or even not to question, and from this mental attitude we cannot extricate ourselves. Before, however, any of the questions can be answered as regards Truth, we must go to the bases of all our separate investigations; that is we must find out in each branch of study those fundamental concepts upon which we had previously built up our intellectual edifice, implicitly to a great extent, and with our ideas of them only gradually becoming more and more distinct. The necessity for this process of analysis is obvious; for, if we knew the strength of our premises, we should know also the force with which the conclusions follow, and so be in a better position to answer the questions relating to Truth in which we are so deeply interested. The need for clarifying our fundamental concepts arises also when we consider that the whole intellectual conquest is based upon them, and that after each momentary advance there will be a setback unless the bases themselves are reinforced and made more secure. This is done when they are made as universal as possible, when they are made so general as to include all our previous concepts and laws. They are rendered indeed the most impregnable when they are derived from the very nature of things.

The study of the bases of the science of mathematics was taken up seriously only in the last century. In this connection our concept of "number" was subjected to a minute analysis so as to obtain a more comprehensive concept in which all our previous ideas were involved. We started with the integers; we introduced the rational numbers, zero and the negative numbers, irrational numbers, transcendental numbers and complex numbers; and each addition was made in such a way that our previous system of numbers became only an incomplete purview of the new system which is the more comprehensive.

The experimental and the inductive sciences all crave for exactitude, and this they attempt to obtain by searching for what are called the "Laws of Nature". We shall refer to these "laws" subsequently. Mathematics, on the other hand, does not become more exact by the additions we have referred to, by the widening of our fundamental concepts. It was just as exact a science a century ago.

190018
JL
Bm211,N33M8

and also a thousand years ago, as it is today. This is because the bases of Mathematics do not depend for their truth upon experience, but are abstractions from the very nature of things and these abstractions have been made at the very beginnings of the science; in fact, without this primary abstraction there could be no mathematics at all. The notions of oneness and of sequence or order are fundamental ones and these have been abstracted from the very nature of things about us. That they are notions or concepts does not detract from the reality of oneness or order; they are "universals" and they have their counterpart in things; and it is the good fortune of this science at its very inception to strike upon universals upon which to build itself.

Let us define "one", in the concrete, as that which is undivided in itself and divided from everything else.

No doubt this circumscription of an existent thing, or being, considering it as a whole and distinguishing it from other things, is only a process of the mind; but this process would not be possible if in the nature of things nothing could exist as itself and distinct from other things. We may consider something as a whole bound together, as it were, by a single character in its entirety. That single character may be its particular *mode of existence* rendering it *that* thing A and A alone; as when we say: Peter is one; or it may be that there is a *common trait*, a notion applicable to things otherwise discreet, as when we place Peter and John in *one* class, say, English; or Peter, John, and Raman in *one* class: man. The more general the common notion binding the individuals, the more comprehensive does the class become. Again we distinguish one thing A from everything else by seeing at least some character in A which is not found in everything else.

We need not go further; for to elucidate the notion of oneness would really amount to tautology. As will be seen, we have used the words "single", "common", "particular", which themselves involve the same notion.

Let us take it for granted then that the concept of unity as oneness, of unity as being bound together and set apart, of uniqueness, of unity as a number arises from the very nature of things, in the same way as our concepts of identity and distinction do.

As a transcendental or attribute predicable about all things, oneness is practically the same as "being", for the trait common to all things is being or existence, which being is perceptibly individuated,

Correlative with the idea of unity is that of multiplicity. It would be moving in a circle to attempt to elucidate this concept with the help of that of unity.

We next come to consider "order" in the sense of a definite sequence. This also is a fundamental concept and ultimately arises from the nature of "Time", out of the influence of which we cannot escape. The sequence of the letters in the alphabet, of the telegraph poles alongside a road, or of any other concrete multiple, can all be referred in some way or another to this continuous flow of time; both because time not only affects all concrete realities, but also because it affects my thinking: I can think of only one thing at a time, that is, of units.

I can mentally put a oneness into a multiple reality, and I can put into it an order also. Away from my mind and independent of it, there is something real which enables me to do so. In this sense oneness and order or sequence are abstractions from the reality, what in Logic are called "universals".

Our system of integral numbers also is an abstraction. The multiple reality is sorted out into ones, and an order placed among them; and what is at least ideally the last in any sequence whatever formed from them corresponds to a "number". In other words, if a multitude of ones be taken, the same number will correspond to the last in whatever order we arrange them. Thus, in the sequence A B, ordered as written, if A corresponds to 1, B is *made* to correspond to 2; and our concept of 1 and 2 is such that, if we had written B A and made B correspond to 1, A would correspond to 2. The same applies to the other numbers. This is possible because there is a oneness in A and B as well as a discreetness; A and B are one in one sense, but distinct in another. *Inasmuch as they are one, the change in the order is immaterial; inasmuch as they are distinct proceeding from A to B we shall have a concept corresponding to B different from that corresponding to A;* this is expressed by the word "two". Expressing this in another way, we may say that we abstract something common from the termini of the sequences A B and B A, and this we call the Number "two". Thus we invent the integers to correspond to a definite position in a sequence of individuals of a multiple reality in every member of which there is a common oneness. Again, because A and B together enjoy a common oneness, and because this oneness is not the same as that by which each is one, we can put the number 1 in correspondence with either A or B, and the number

2 with them taken both together. The same applies to the higher numbers. This is the basis of our concept of *quantity* which can be put into correspondence with number in the same way as any sequence of ones. We shall deal with the concept of quantity in another article.

Our integral number-system is therefore "ideal", but it is an abstraction from the nature of things, and, consequently, the properties of numbers arise from the very nature of things. They have not been empirically established. Their relations are invariant. To take an example, whether we speak of apples or oranges or monkeys two and two make four; for note that when we apply the numbers to the concrete, it is only the oneness of the members in some one respect, and their discreetness in other respects, and the possibility of their being arrayed in an order, that enables us to do so.

Two apples and two oranges make four fruit. The word "apples" denotes the unity in the members of the first group; and the word "oranges" shows that the members of the second group are one in a certain respect; whereas the word "fruit" shows a unity in both the groups.

In the statement: "one drop of water added to one drop of water makes one drop of water", the unity in each of the entities referred to has regard to a common mode of physical existence; whereas in the statement: "one drop of water added to one drop of water makes two drops of water", the "two" does not refer to the physical mode of existence as the "one" and the "one" may do, but all the numbers 1, 1 and 2 are applied quantitatively. On the other hand, if we say that they make three drops of water, then the "three" refers to a possible division into three units, each of which, however, is not a unit in the same sense as each drop in the subject of the sentence is one.

Numbers are not in things; it is our mind that puts them into things. *Just as our minds abstracted them from things so in the same way and exactly by the reverse operation and with the same limitations, we can apply them to things.*

"(Mathematical) principles are seen to be intrinsically necessary as soon as they are understood; whereas inductive conclusions are never seen to be intrinsically necessary, but only to be unavoidable. Nor does further experience add anything to our assurance........" (Joseph: "An Introd. to Logic" 2nd ed. pp. 551-'2) "we must

conclude that in mathematics there is (or at least should be) no generalization from experience. To suppose mathematical principles to be such generalizations is like supposing the Law of Causation to be so ...... There can be no knowledge unless there are truths not empirical—i.e. not open questions, for a decision on which we must go to the tribunal of sense-perception or events. And no one will understand the structure of mathematics, who does not see that mathematical principles are truths of this kind." (*Ibid.*)

To the integral system of numbers, the fractions were added making up what is called the rational semi-continuum. With the introduction of the irrationals, of zero and the negative numbers the whole arithmetic continuum of numbers was built up, based primarily on the system of integers. It is remarkable that the notion of zero as a number came very late in history. This is not strange however, for number is primarily an abstraction from things real, so that zero as a number could not be associated with anything real, since it could only be associated with nothing, and "nothing" is not real. It was only when it was realised that the number-system could be shifted about in its application to concrete objects that zero and the negative numbers became a recognized part of the number-system. Some harmoniums are provided with a movable keyboard to enable a beginner to accompany voices by playing from a score without mentally having to transpose into another key suitable to the scope of the singers. In the same way it is with the number-system.

The Laws of Arithmetic follow necessarily from the number-system. These Laws are so called not in the same sense in which we use the term in connection with the "empirical laws", to which by a strange mischance the expression "Laws of Nature" was given. The latter were set up as "Laws of Nature" by the obstinate determinism of the last century. The laws of arithmetic and those by which we can legitimately bring into correspondence material objects, quantity and position, with our number-system, have a distinct right to be classed under the term "Laws of Nature"; for as we have seen they arise from the very "nature" of things and cannot be brought into question or doubted; nor have they been obtained inductively or experimentally by an exhaustive repetition of observations.

The notion of "continuity" in the number-system, in the technical sense, was obtained even before the recognition of surds as numbers. Moreover, although the numbers, either integer or rationals, or reals, are not enumerable, yet we know the laws which all obey.

The terms with which we deal are not "unique"; considered as a whole they are homogeneous. Consequently, when we give a proof by "mathematical induction", the proof is strictly valid, no repeated experience or exhaustive tabulation of results being necessary as in the case with induction strictly speaking.

Consequently also we can generalise still further, and introduce general symbols, and find invariant relations and invariant "forms." The statement of the permanence of Equivalent Forms is only that of the invariant character of many relations between numbers obtained from their generalised concept. A great deal of the mathematician's time is spent in the search for invariants. Thus is built up the science of Algebra. This science, purporting primarily to deal with the specific properties of numbers would involve us in difficulties: we would arrive at a stage when we should either acknowledge exceptions to certain consequences which we had discovered, or choose as an alternative the introduction of complex quantities in such a way that the previous number-system was only a particular case of the generalised system of dual numbers now introduced. This done, our power of advancing further is renewed. Note, however, that the extension is due only to our desire that a certain consequence should hold in general; e.g., that every equation should have a root, that every equation should have as many roots as indicated by its degree. If we refused to extend the number-concept, we could not make these statements as general ones; that is all. Considered in itself the number-concept would still not be defective. In the same way and for the same reason, operative symbols are found to require a more and more comprehensive interpretation, e.g., exponentials: a^x is given a meaning for any value whatever of x of our completed number-system. Each extended interpretation involves as a particular case the previous one. Extended meanings may also be assigned to functional symbols. In fact, generalising the methodology itself of mathematics, we can invent as many functions as we choose, provided we make any one of them consistent with itself and obedient to certain specific laws, which laws themselves should be consistent with the general properties of numbers. Further, we might introduce functions and their laws by analogy, and investigate the consequences. These may be of practical value if, in the course of investigation, we strike upon a relation between the function started with and some function already known. All direct processes are of course most useful in helping us in the no less important reverse processes; for example, in finding a function, or an invariant form, with certain particular properties.

The continuity in the system of real numbers leads to the notion of a "limit", and the theory associated therewith becomes fundamental in most of the higher branches of mathematics. Thus the whole of mathematics depends fundamentally and ultimately on the number-concept.

In the present article the writer has not considered Geometry and our concept of Space, or the more elusive concept of time. Each of these can certainly be put into relation with the number-concept; each, however, has a peculiarity of its own which may not be set aside by our mind, but has to be considered as being intimately associated with the number-aspect of it.

We propose in a subsequent article to deal also with the application of mathematics to the investigation of physical laws and theories, and of the nature of the Universe in general.

THE BACKWARDNESS OF EARLY AMERICAN MATHEMATICS

BY

G. A. MILLER, *University of Illinois.*

The pioneering days of exploration and settlement in America were also pioneering days in the development of new mathematical subjects in Europe, including the very fertile fields of analytic geometry and the calculus. The former pioneers were however entirely distinct from the latter which include such now widely known men as R. Descartes, I. Newton, G. W. Leibniz, the Bernoullis, L. Euler, and J. L. Lagrange. The sixteenth, seventeenth and eighteenth centuries witnessed great advances in mathematics in Europe while it has been properly said that in America they were "barren of achievement in the domain of mathematics." *The contradictory assertion contained in the same book that "it is not in the schools that the advance in mathematics in the sixteenth, seventeenth, and eighteenth centuries in America is to be found, but rather in the quasi-astronomical work of the astrologers and the makers of almanacs"* seems to be due to a slip since there were then no such advances in our country.

* D E. Smith and J. Ginsburg: *A history of Mathematics in America before 1900,* pages 10 and 65.

The backwardness of early mathematics in America is especially noticeable in research. With respect to organization and exposition the lag is less marked. For instance, the New York Mathematical Society, which became the American Mathematical Society in 1894, was organized in 1888. This is only twenty-three years after the organization of the London Mathematical Society, which is the earliest national mathematical society of Europe. The corresponding society in Germany, Deutsche Mathematiker-Vereinigung, was organized in 1890—two years after the New York Mathematical Society was organized. The first permanent mathematical periodical in America was started under the name *Analyst* in 1874, and the *American Journal of Mathematics* was started four years later, while the earliest permanent mathematical journal in Europe, the *Journal de l'Ecole Polytechnique*, was started in 1795, and the purely permanent mathematical periodicals were inaugurated in Europe more than a quarter of a century later.

Many of our early professors of mathematics paid too much attention to the writing of text-books but in most cases these text-books exhibit little originality and many of them were based very largely on those of foreign countries. W. Chauvenet, who was one of the incorporators of the National Academy of Sciences of the United States of America (1863), was especially successful in exposition. In 1926 the Mathematical Association of America established a prize, named "Chauvenet Prize" for the best expository mathematical paper published in English during successive periods of five years by a member of this Association. It thus recognized in a significant manner the work of W. Chauvenet and the importance of good expositions, but from the standpoint of research the greatest bane of early mathematics in America was the proclivity of the professors of this subject in our colleges and universities to write very elementary text-books.

An American writer, Jared Mansfield (1759-1832), published an undated volume entitled *Essays, Mathematical and Physical* (about 1801) in which the first 18 pages are devoted to the theory of negative numbers, but this theory is not useful and it was not adopted by the later writers. In fact, it is here stated, page 4, that "it would be impossible by any reasoning to determine the product of $-a$ multiplied by $-b$, abstractly considered." Although this volume has no mathematical value, it is said to have attracted the attention of Thomas Jefferson, who was then President of the United States of America and who rewarded its author by appointing him Captain of Engineers

in the United States army (1802) and in the following year Surveyor-General of the United States. From 1802 to 1803 he was also acting professor of mathematics in the United States Military Academy at West Point, and from 1812 to 1828 he was professor of natural and experimental philosophy in the same institution.

The fact that a relatively prominent American mathematician, who was also a graduate of Yale College, had a very imperfect knowledge of the theory of negative numbers near the beginning of the nineteenth century is partly explained by noting that even in Europe a correct theory of these numbers was then new and not widely known. It is, however, worth observing that American efforts to supply a satisfactory theory of these numbers contributed nothing towards their establishment notwithstanding the elementary nature of such a theory. The reception with which the theory noted in the preceding paragraph met in America is illustrated by the following remark in the earliest American mathematical periodical. "Jared's discourse on this subject reminds me of a Connecticut carle, who said that the moon was exactly the size of a johnny-cake." *Mathematical Correspondent*, volume 1 (1804), page 124. This remark naturally reacted unfavourably on this periodical which was discontinued soon thereafter.

From the manner in which the President of the United States of America rewarded the feeble efforts of J. Mansfield to contribute to the advancement of mathematics it seems that the backwardness of early mathematical research in America was not due to a lack of appreciation on the part of the American public, as has sometimes been claimed. J. L. Coolidge pointed out in his address entitled "Robert Adrian and the beginning of American mathematics," *American Mathematical Monthly*, volume 33 (1926), pages 61–76, that the more successful efforts of R. Adrian (1775–1843) along the same line were also highly rewarded by his American contemporaries. The professorships of mathematics in our early colleges were filled by men who were not distinguished for research because such men were not then available. When the University of Virginia was founded Thomas Jefferson urged the appointment of foreigners to most of the professorships in this institution including that of mathematics, because suitable men were then too scarce in our country.*

If by history of mathematics we mean the history of effective contributions towards the development of this subject it is questionable whether American work prior to the last quarter of the

* Educational Works of Thomas Jefferson, page 96, 1931.

nineteenth century constitutes a part of this history. It is true that some independent discoveries in this subject were made before this time but they are not known to have then affected the later developments made in Europe along the same line. In particular, while the work of B. Peirce on linear associative algebra attracted prompt attention in Europe, no review thereof appeared in the *Jahrbuch über die Fortschritte der Mathematik* although such a review was promised by F. Meyer in the volume of this series for 1881, page 82. This work by B. Peirce has, however, the distinction of being the first known attempt to classify systems of complex numbers and hence it occupies an important place in the history of mathematics in general as well as in the history of American mathematics.

In England H. Briggs, who published in 1617 the first modern table of logarithms and hence should be regarded as one of the founders of this useful subject, has the singular distinction of holding in succession the two earliest chairs of mathematics that were founded in that country. The first of these was established at Gresham College, London, in 1596, and the second at Oxford in 1619. In Cambridge, England, the first chair of mathematics was filled in 1662 by the appointment of I. Barrow who was succeeded seven years later by I. Newton. Hence some of the leading professorships of mathematics in England were filled from the beginning by research men while in America about a century and a half elapsed from the time of the appointment of a professor of mathematics at William and Mary college in 1711 until an incumbent of such a position made any definite direct contribution towards the advancement of this subject, as noted in the preceding paragraph. Most of the earlier work by R. Adrian seems to have remained ineffective until it was rediscovered independently in Europe.

GLEANINGS

36. He who guides the comets with his hand,
He who holds the stellar nebulæ,

Shall he not my life's chart understand,
Shall he not guide me.

E'en the human mathematiker
Can compute the farthest decimal ;

So the great exact Astronomer,
He who wakes while mortals sleep—

In its orbit he will keep,
Mortal man's infinitesimal.

Mary Willard Gleason White (From *Scripta Mathematica* March, 1933.)

THE CORRECTIONS OF 'VARIATION' AND 'EVECTION' IN HINDU ASTRONOMY

BY

D. AKKU SOMAYAJULU, *Research Scholar, Madras University.*

The process of obtaining the true places of the planets from the mean ones, is called *Grahasphutam* ग्रहस्फुटं in Hindu Astronomy. The method adopted by the Hindus is based upon the theory of epicycles. The Sun and the Moon are rectified (*i.e.* their true positions are obtained from the mean ones) by the single process called *Mandakarma* मांदकर्म while the other planets are rectified by the true processes called respectively the मांदकर्म and शैघ्रयकर्म (*Saighryakarma*).

"चन्द्रसूर्यौ स्फुटौ स्यातां मान्देनैकेन कर्मणा"

The reason is that, the geocentric position of the Sun and of the Moon can be got by applying a simple process corresponding to motion about the earth, while in the case of the other planets, their heliocentric positions have first to be got, and from these the geocentric positions. The former process, that is, the process by which their heliocentric positions are obtained, is called the *Mandakarma*, since it involves only the same principles as in the case of the Sun and the Moon; and the second step, that is, the one by which their geocentric positions are obtained from the heliocentric ones, is called शैघ्रयकर्म *Saighryakarma*. We shall, in this paper, concern ourselves mainly with the motion of the Moon.

In modern Astronomy, the Moon's position is calculated in the following manner:—

The equations of motion according to Depontecoulant are

$$\text{(i)} \quad \frac{1}{2}\frac{d^2}{dt^2}(r^2) - \frac{\mu}{r} + \frac{\mu}{a} = r\frac{\partial \text{R}}{\partial r} + 2\int \frac{d'\text{R}}{dt}\,dt$$

$$\text{(ii)} \quad \dot{v} - \frac{h(1+s^2)}{r^2} = \frac{1+s^2}{r^2}\int \frac{\partial \text{R}}{\partial v}\,dt$$

$$\text{(iii)} \quad \frac{d^2 z}{dt^2} = \frac{\partial}{\partial z}\left(\frac{\mu}{r} + \text{R}\right) \quad \text{or} \quad \frac{d^2 z}{dt^2} + \frac{\mu z}{r^3} = \frac{\partial \text{R}}{\partial z}$$

where R, the disturbing function due to the attraction of the Sun is given by

$$R = m'\left(\frac{r^2}{r'^3}p^2 + \frac{r^3}{r'^4}p^3 + \cdots\right) \quad \text{(Brown \textit{Lunar theory} p. 16.)}$$

To solve these equations of motion, an intermediate orbit is chosen which is a close approximation to the actual orbit, and the small corrections that have to be made in the co-ordinates afterwards are found. The intermediate orbit chosen is, of course, the simple ellipse obtained when the Sun's action is altogether neglected, in which case, the co-ordinates have the values

$$\frac{r}{a} = 1 - e\cos\omega + \tfrac{1}{2}e^2(1-\cos 2\omega) + \cdots\cdots$$

$$v = nt + \varepsilon + 2e\sin\omega + \tfrac{5}{4}e^2\sin 2\omega - \tfrac{1}{4}e^3\sin 3\omega + \cdots\cdots$$

where ω = the mean anomaly $= nt + \varepsilon - \omega$.

The corrections to be made afterwards are (i) the Variation, (ii) the Evection, (iii) Parallactic inequality, (iv) the Annual Equation, which arise respectively when we

(i) consider only R (the disturbing function) and neglect e, e', a/a', and γ.*

(ii) include terms depending on the first power of e and neglect e', a/a', γ.

(iii) consider first powers of a/a' and neglect e, e', r.

(iv) consider e' and neglect others.

In these corrections, the Variation and the Evection are the only big corrections and the others two are inconsiderable. The "Variation" is given by the two equations.

$$\text{(i)} \quad \frac{a}{r} = 1 + \frac{1}{6}m^2 + \left(m^2 + \frac{19}{6}m^3\right)\cos 2\xi.$$

$$\text{(ii)} \quad v - v' = \xi + \left(\frac{11}{8}m^2 + \frac{59}{12}m^3\right)\sin 2\xi.$$

where $m = \dfrac{n'}{n} = \dfrac{\text{sidereal month}}{\text{sidereal year}} = \dfrac{1}{13}$ approximately and ξ is the difference of the mean longitudes of the sun and the moon.

---

* e = Eccentricity of the moon's orbit round the Earth.
e' = " of the Earth's orbit round the sun.
Moon's mean diameter/Earths' mean diameter.
and $\gamma = \tan i = \tan$ (inclination of the moon's orbital plane to the plane of the Ecliptic.)

Also the correction of Evection is given by

(i) $\frac{a}{r} = \left(1 - \frac{7}{12}m^2\right)e \cos \phi + \ldots .$

(ii) $(v - v') = \frac{15}{4}\, me \sin (2\xi - \phi) + \ldots\ldots$

In the case of 'Variation' the correction in longitude namely $\left(\frac{11}{8}\, m^2 + \frac{59}{12}m^3\right)\sin 2\xi$ comes to $\sin 2\xi \times \left(\frac{11}{8} \times \frac{1}{13^2} + \frac{59}{12} \times \frac{1}{13^3}\right)$ radians roughly, or since a radian is 3438 seconds of arc, to

$$\left(\frac{11}{8} \times \frac{1}{13^2} + \frac{59}{12} \times \frac{1}{13^3}\right) \times 3438 \times 60'' \times \sin 2\xi = 2145'' \sin . 2\xi$$

approximately.

Calculating the variation for various values of the argument ξ (mean Elongation of the moon) it can be tabulated thus:—

Table No. 1

ξ in degrees	Variational correction in minutes	ξ	Variational correction in minutes	ξ	Variational correction in minutes
0°	0	35°	32·5′	70°	20·9′
5°	6·1′	40°	33·9′	75°	15·7′
10°	11·9′	45°	34·3′	80°	10·1′
15°	17·5′	50°	33·6′	85°	4·1′
20°	22·4	55°	31·8′	90°	− 2·1′
25°	26·6′	60°	29·0′		
30°	30·1′	65°	25·4′		

Now in Hindu Astronomy, the intermediate orbit is obtained more or less with a fair amount of accuracy by the application of epicycles; but since there will be an obvious difference between the moon in the intermediate orbit and the true planet which can be obtained only after the further corrections specified above, the Hindu Astronomers named the further correction which achieves दृक्साम्य *Drik-samyam* for the Moon (i.e. equality with the observed moon) as बीजं *Beejam* Bhaskara gave a definite shape to this *Beejam* classifying it into two kinds, the first of which exactly corresponds to *Variation*, and the second to the *Evection*.

He got the magnitudes, as he says, from accurate observations only and insisted upon the adoption of this *beeja* to achieve *Driksamyam.* Many Siddhantis today are under the misapprehension that बीजसंस्कार is only arbitrary, and should not be introduced into the calculation of तिथि *thithi*, though it may be introduced into the calculation of the eclipses, as they say :—

प्रत्यहं तिथिनक्षत्रयोगस्यानयने विधुः ।
अबीजसंस्कृतो ग्राह्यो ग्रहणादौ सबीजकः ॥

In their opinion, ceremonies which are conducted according to *thithis*, should not be subject to a vague and arbitrary correction called *beejam.* Even Mr. Dattaraju Ketkar, the son of the late Mr. V. B. Ketkar, says in his *Ketakigrahaganitam* that *beeja* is different from *Samskara* the difference between them being that the former is arbitrary and unscientific, while the latter is Scientific. I do not know, which exactly he means by *beeja* here, but if he has in his mind the *beeja* of Bhaskara, whom he very often quotes and reveres, I must say, that he too is mistaken like the other Siddhantis in saying that the *beeja* of Bhaskara is arbitrary and unscientific "अनुपपत्तिमत्". It might be that Bhaskara could not have known the exact basis of the *beeja*, as we do now with the help of Dynamical Astronomy, but he was perfectly correct in his assigning the magnitudes of the *beeja* for a given argument. This reflects great credit to his genius in discovering an inequality, which could not be discovered but with the help of a powerful instrument like the Dynamical Astronomy.

In the following part of the paper, let us see how Bhaskara's second बीजसंस्कार is not at all arbitrary as thought to be, but corresponds exactly to the variational correction and his first one to that of evection.*

Bhaskara's statement in his work called बीजोपनय (*Bijopanaya*) is to quote his own words

1. "अर्कस्फुटाच्चन्द्रमिमं विशोध्य
शिष्टे ऋणं स्वोजपदे फलं स्यात् ।
अतोऽन्यथान्यत्र यथाक्रमं वै
ब्रुवे फलानामपि पिण्डकानि ॥"

* The late Mr. V. B. Ketkar, no doubt, recognises that the totality of the inequalities in Modern Astronomy, is equal to the totality of those in Hindu Astronomy, but fails to see that the बीज are the Variational and Evectional corrections. (Ref. Mr. Ketkar's *Indian Calendar.*)

2. रसाश्च नन्दा गुणतारकेशो
भूभृद्भवौ बाहुकरौ जिनाश्च ।
ताराः खरामा द्विगुणाश्च देवा
वाराशिरामाः सरिदीशकालाः ॥

3. वेदाग्नयो दानवशत्रवश्च शशाङ्कवह्नी नवबाहवश्च
रसाश्विनौ वेदकरौ खबाहू रसक्षमे रुद्रगजा अनलाः खम् ।
अनेन युक्तश्च शशीस्फुटः स्यात्
कर्मार्हकालानयनोपयोग्यः ॥

i.e. "Subtract the moon from the Sun and have the convention that, if the remainder lies between '0' and 90° or between 180° and 270°, then the correction is negative; otherwise it is positive; hereunder I give the various values of the *beeja* for various values of the argument ($2\pi - \xi$) here"; and he gives them in the 2nd and 3rd slokas. They can be tabulated thus:—

Table No. 2

Argument $2\pi - \xi$	*Beeja*	$2\pi - \xi$	*Beeja*	$2\pi - \xi$	*Beeja*
3¾°	6′	33¾°	32′	63¾°	26′
7½°	9′	37½°	33′	67½°	24′
11¼°	13′	41¼°	34′	71¼°	20′
15°	17′	45°	34′	75°	16′
18¾°	22′	48¾°	34′	78¾°	11′
22½°	24′	52½°	33′	82½°	8′
26¼°	27′	56¼°	31′	86¼°	3′
30°	30′	60°	29′	90°	0′

His statement that the *beeja* is negative in the odd quadrants and positive in the even, is justified since his argument is $2\pi - \xi$ and not ξ.

Now, calculating this *beeja* corrections for arguments 5°, 10°, etc. so as to compare this with the Variational correction, they can be tabulated thus :—

Table No. 3

ξ	*Beeja*	ξ	*Beeja*	ξ	*Beeja*
5°	7′	35°	32·3′	65°	25·3′
10°	11·7′	40°	33·7′	70°	21·3′
15°	17′	45°	34′	75°	16′
20°	22·7′	50°	33·7′	80°	10′
25°	26′	55°	31·7′	85°	4·7
30°	30′	60°	29′	90°	0′

Comparing this with Table No. 1, we see a remarkable agreement between the two, the difference never exceeding ·6 of a minute of arc except when the argument is 90° when the difference is 2′ of arc.

As regards *Evection* the correction in longitude, is $\frac{15}{4}$ *me* sin $(2\xi - \phi)$ approximately and when calculated correctly taking $e = \cdot 0549$, comes to $4467'' \sin(2\xi - \phi)$.

Calculating the Evection for various values of the argument $(2\xi - \phi)$ i.e. $2 \times$ moon's elongation — moon's mean anomaly [i.e. 2(चन्द्रः—सूर्यः)—चन्द्रमन्दकेन्द्रं] it can be tabulated thus :—

Table No. 4

$2\xi - \phi°$	Evection	$2\xi - \phi°$	Evection	$2\xi - \phi°$	Evection
5°	6·4′	35°	42·1′	65°	66·9′
10°	12·7′	40°	47·3′	70°	69·5′
15°	18·9′	45°	52·1′	75°	71·5′
20°	25·0′	50°	53·4′	80°	73·0′
25°	31·0′	55°	60·4′	85°	74·1′
30°	36·6′	60°	63·9′	90°	74·3′

Bhaskara's first *beeja* is given in the following table, calculated to the same series of arguments as above :—

Table No. 5

Argument	*Beeja*	Argument	*Beeja*	Argument	*Beeja*
5°	8·3′	35°	57·7′	65°	75·3′
10°	18·3′	40°	63·7′	70°	76·7′
15°	27′	45°	68′	75°	77′
20°	34·7′	50°	70·7′	80°	78′
25°	42·7′	55°	73·3′	85°	78′
30°	51′	60°	75′	90°	78′

Hence comparing this table with Table No. 4, we find that the divergence is at most equal to 18′ of arc. Even this divergence must have arisen, in my opinion, due to Bhaskara's probably mistaking * the argument to be ξ instead of $2\xi - \phi$.

That Bhaskara means only Evection in this Beeja is also justified from his statement in *Bijopanaya* (*cf.* Slokas 20—25) that (i) the greatest subtractive difference occurs between the calculated and observed moon's place when the sun remains at a distance of 180° from the moon placed at the end of 90° from its *Thungsthana* तुङ्गस्थान (higher apsis); (ii) that the greatest additive difference occurs when the sun remains at a distance of 180° from the moon situated at 270° at the end of the 3rd quadrant from its higher apsis ; (iii) that there is no beeja, when the moon and the sun reside together in the *Thunga* or *Neecha* (lower apsis) of the moon. Here, we cannot say that Bhaskara might be referring to *Variation*, for, in nothing else except Evectional correction do we require the *Thunga* besides the sun and moon, not even in Variation. The meaning of Bhaskara's statement above is clear if we put it in symbols :—that is (i) if $\xi = 180°$ and $\phi = 90°$ then $\sin(2\xi - \phi) = \sin(360-90)° =$ a negative maximum. (ii) if $\xi = 180°$ and $\phi = 270°$ $\sin(2\xi - \phi)$ is a postiive max. (iii) when $\xi = 0$ and $\phi = 180°$ or 0°

* I request to be excused for saying that Bhaskara was 'probably' mistaken ; I have my grounds in the following argument above. Or, since the text is vague here, it is also possible to interpret that Bhaskara meant only $2\xi - \phi$ and not ξ from the same argument.

then sin $2\xi - \phi$ is zero. Thus it is obvious that Bhaskara means only Evection and nothing else in this *Beeja*. It may be asserted more definitely than otherwise, that Bhaskara meant the argument also to be $2\xi - \phi$ from his citing in all the three cases he quoted, the relative positions of the sun, moon and the apses.

ON THE NATURE OF THE ROOTS OF CERTAIN EQUATIONS

BY

M. N. NARASIMHA IYENGAR, *Bangalore*.

In a note in Vol. I, No. 1 of the "Mathematics Student", Dr. C. N. Srinivasiengar discusses in detail the nature of the roots of the equation $\phi(x) \equiv f(x) + xf'(x) = 0$, where

$$f(x) = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^r}{r!} \tag{1}$$

when r is odd. He has not however discussed the case when r is even, beyond pointing out that when $r \leqslant 6$, the roots need not be all imaginary. The main object of this paper is to discuss this case in full. Incidentally, a somewhat shorter proof is given to show that when r is odd, $\phi(x) = 0$ has only one real root, which lies between 0 and -1.

1. To prove that $\phi(x) = 1 + \frac{2x}{1!} + \frac{3x^2}{2!} + \ldots\ldots + \frac{(r+1)x^r}{r!} = 0$ has one and only one real root when r is odd, and two and only two real roots when r is even except for $r = 2$ and $r = 4$ in which case all the roots are imaginary.

Proof:—$\phi(x) = f(x) + xf'(x) = (1+x)f'(x) + x^r/r! \quad \cdots\cdots \quad (2)$

$= (1+x)f(x) - x^{r+1}/r! \quad \cdots\cdots \quad (3)$

where $f(x)$ is given by (1).

Suppose r is odd. In this case $f'(x) > 0$ for all real x. By (2) $\phi(x) < 0$ for $x \leqslant -1$. Hence $\phi(x) = 0$ cannot have any real root less than or equal to -1. Again, we get

$$\phi'(x) = (1+x)f'(x) + f(x) - \frac{r+1}{r!}x^r = (2+x)f'(x) - \frac{x^r}{(r-1)!}$$

$\therefore \quad \phi'(x) > 0$ for $-1 < x < 0$.

i.e. $\phi(x)$ is monotonic increasing in the range $(-1, 0)$, from $-1/r\,!$ to $+1$. Hence there is one and only one real root in the interval $(-1, 0)$.

This real root is not greater than $-\frac{1}{2}$ since $\phi(-\frac{1}{2}) > 0$, as may be easily verified.

This may also be deduced from Prof. Kakeya's result * that the absolute values of the roots of the polynomial $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0 = 0$ (all coefficients > 0) lie between the greatest and the least values of the quotients $a_{n-1}/a_n, a_{n-2}/a_{n-1}, \cdots a_0/a_1$;

$\therefore$ $\phi(x)=0$ has one and only one real root when r is odd and that root lies in the interval $(-1, -\frac{1}{2})$.

2. *Suppose r is even* :—$f(x) > 0$ for all real values of x. By (3), $\phi(x) > 0$ for $-1 < x < 0$ and $\phi(x) > 0$ for $x \geqslant 0$ $\therefore \phi(x)=0$ has no real root greater than or equal to -1.

It is well known that $f'(x)$ is monotonic increasing when r is even and increases from $-\infty$ to $+\infty$. Suppose α_1 is the real root of $f'(x)=0$. Then $\qquad f'(x) < 0$ for $x < \alpha_1 \qquad \ldots(4)$

$\therefore$ $\phi(x)=f(x)+x f'(x) > 0$ for $x < \alpha_1$ where $f'(a_1)=0$.

$\therefore$ There is no real root of $\phi(x)=0$ less than or equal to α_1; i.e. all the real roots of $\phi(x)=0$ lie between α_1 and -1. It is obvious from Prof. Kakeya's result that $\alpha_1 > -(r-1)$.

$$\phi'(x)=(1+x)\ f'(x)+f(x)-\frac{r+1}{r\,!}x^r=(2+x)f(x)-x^r\ (x+r+2)/r\,!$$

Hence $\phi'(x)$ is negative for $-2 > x > -(r+2)$ since $f(x) > 0$; i.e.

$\phi(x)$ is decreasing in the interval $-(r+2) < x < -2$

Now $f'(-r-2) < 0$ by (4) $\therefore \phi(-r-2)=f(-r-2)-(r+2)f'(-r-2) > 0$

Again, by (2), $\phi(-2)=-f'(-2)+2^r/r\,! > 0$ when $r=2$ and $r=4$ as may be easily verified.

Therefore there cannot be any root of $\phi(x)=0$ in the interval $\{-(r+2), -2\}$ when $r=2$ and $r=4$. When $r \geqslant 6$,

$$-\phi(-2)=\{f'(-2)-2^r/r\,!\}=\{f(-2)-2^{r+1}/r\,!\} > \frac{1}{e^2}-\frac{2^{r+1}}{r\,!}$$

which is positive for $r \geqslant 8$ and it can be seen by actual verification that $\phi(-2) < 0$ for $r=6$.

* Tohoku Math. Journal Vol. 2. pp. 140-142.

Thus we have shown that

(i) $\phi(x)$ is monotonic decreasing in the interval $-(r+2)<x<-2$

(ii) $\phi(-r-2)>0$ and $\phi(-2)>0$ for $r=2$ and $r=4$

(iii) $\phi(-r-2)>0$ and $\phi(-2)<0$ for $r\geqslant 6$

Hence $\phi(x)=0$ has no real root less than or equal to -2 when $r=2$ or 4 and has one and only one real root in the interval $(\alpha_1, -2)$ when $r\geqslant 6$, r being taken to be an even number.

3. Next we consider the interval $(-2, -1)$. We have

$$\phi''(x)=(3+x)f''(x)-\frac{x^{r-1}}{(r-2!)}.$$

Since $f''(x)>0$ for all x, $\phi''(x)>0$ for $-2<x<-1$.

$\therefore$ $\phi'(x)=0$ cannot have more than one real root and $\phi(x)=0$ not more than two real roots in the interval. But when $r\geqslant 6$ there is one and only one real root less than -2 $\therefore$ $\phi(x)=0$ has one and only one real root in $(-2, -1)$ when $r\geqslant 6$. It may be easily seen that $\phi(x)=0$ has no real root in the range $-2<x<1$ when $r=2$ and $r=4$. Thus

i. $\phi(x)=0$ has one and only one real root when r is odd and that root lies in the interval $(-1, -\frac{1}{2})$.

ii. $\phi(x)=0$ has no real root when $r=2$ and $r=4$.

iii. $\phi(x)=0$ has two and only two real roots when r is even and $\geqslant 6$. Of these two roots, one is in the interval $(\alpha_1, -2)$ and the other in $(-2, -1)$.

More generally it may be shown that if $\phi(x)=f(x)+x^n f'(x)$. where n is odd and r is also odd, then $\phi(x)=0$ has one and only one real root and that lies in the interval $(-1, 0)$.

In conclusion, I desire to state that almost all equations of this type may be solved if the general problem *viz.*, "If

$$a_0+a_1\ x+a_2\ x^2+\ldots+a_n\ x^n=0$$

has p real roots and

$$b_0+b_1\ x+b_2x^2+\ldots+b_m\ x^m=0$$

$(m<n)$ has q real roots, find the number of real roots of the equation

$$a_0\ b_0+a_1\ b_1\ x+a_2\ b_2\ x^2+\qquad+a_m\ b_m\ x^m=0"$$

is solved. The above problem has been solved* when all the roots of both the equations are real. In this case the third equation has all

* E. Malo *Journal de Math. Spéciales* (4) Vol. 4, 1895.

JL

190018

its roots real provided the first equation has all its roots real and of the same sign. Anothor similar problem solved* is that if a polynomial $a_0 + a_1 x + \dots + a_n x^n$ of even degree has all its roots imaginary then the equation $a_0 b_0 + a_1 b_1 x + \dots + a_n b_n x^n = 0$ has all the roots imaginary if

$$b_n = \int_e^d f(t)\, t^n \, dt \quad (n = 1, 2, \dots)$$

where c and d are arbitrary fixed real limits independent of n, and $f(t)$ is an arbitrary real function of invariable sign between c and d. But I have not seen the solution of the general problem.

Remarks by C. N. Srinivasiengar

The following proof will suffice to show that when r is even, $\phi(x) = 0$ has two and only two real roots.

If possible, let there be three real roots a_1, a_2, a_3 $(a_1 < a_2 < a_3 < 0)$. We have

$$\phi'(a_s) = f'(a_s) - (r+1)a_s^r / r! \qquad (s = 1, 2, 3).$$

By Lemma IV. (Vol. I, No. 1, p. 26), $f'(x) - (r+1)x^r/r! = 0$ has only one negative root, θ say, and by Lemma I, $\phi'(a_1)$ and $\phi'(a_2)$ are of opposite signs. θ thus lies between a_1 and a_2; hence $\phi'(a_2)$ and $\phi'(a_3)$ are of the same sign, which is impossible.

Next, consider
$$\psi(x) = \phi(x) + \left\{ \frac{(r+2)x^{r+1}}{(r+1)!} + \frac{(r+3)x^{r+2}}{(r+2)!} \right\}$$
$$= \phi(x) + \frac{x^{r+1}}{(r+1)!} \left(r + 2 + \frac{r+3}{r+2}\, x \right)$$

If x be given such a value that $-(r-1) < \alpha < x < -1$. (using Kakeya's theorem), α being the real root of $f'(x) = 0$, and if $\phi(x)$ be negative, $\psi(x)$ is also negative. $\psi(x) = 0$ must therefore possess a real root. Thus, if for a certain even value of r, $\phi(x) = 0$ has two real roots, the same is true for the next even value. But it is directly verified that this is so when $r = 6$.

* M. Petrovitch: *Bull de la Société Math.* Vol. 41 (1913) pp. 194—206.

NOTES AND DISCUSSIONS

The Editor welcomes for publication under this heading, brief discussions of interesting problems, critical comments, and suggestions likely to be helpful in the class-room.

On Sums of like Powers

In this paper I follow the notation of my recent papers.[1] The following are some of the results contained in this paper.

We have*

$$(19,\ 19,\ 15,\ 6,\ 6,\ 2)^7 = (20,\ 18,\ 13,\ 8,\ 8)^7$$

$$(19,\ 19,\ 19,\ 16,\ 6,\ 6)^7 = (20,\ 18,\ 18,\ 17,\ 12)^7$$

which examples prove $\beta(7) \leqslant 6$ ... (1)

Again we have

$$(18,\ 17,\ 15,\ 12,\ 2)^7 = (19,\ 16,\ 13,\ 8,\ 8)^7$$

$$(22,\ 20,\ 9,\ 7,\ 7)^7 = (23,\ 16,\ 14,\ 8,\ 4)^7$$

which examples prove $(5) \overset{7}{=} (5)$ ... (2)

The identity

$$(7x^7+y^7)^7 + (y^7-7x^7)^7 + 2(7x^4y^3)^7 + (6x^4y^3)^7 + (3x^4y^3)^7 + (x^4y^3)^7 + (2x^2y^5)^7 + (x^2y^5)^7$$
$$= 2(7x^6y)^7 + (8x^4y^3)^7 + (2x^4y^3)^7 + (3x^2y^5)^7 + 2(y^7)^7,$$

with $y^7 > 7x^7$, proves $(9) \overset{7}{=} (9)$ *i. o,*

that is $\gamma(7) \leqslant 9$ ... (3)

We have

$$5^9 = 7 \cdot 4^9 + 6 \cdot 3^9 + 19 \cdot 1^9$$

which example proves $(1) \overset{9}{=} (32)$ ... (4)

A fuller version of the paper is to appear elsewhere.

K. SUBBA RAO

---

* $(a,\ b,\ c\ldots)^n$ stands for $a^n + b^n + \ldots$

1 "On equal sums of like powers", *Math. Zeit.* 39, 1934, 240—3. See also, K. Subba Rao "On sums of fifth powers", *Jour. L. M. S.*, 9, 1934, 172—3 and S. Sastry "On sums of powers", *Jour. L. M. S.*, 9, 1934, 242—6.

On the Equation $x_1^n + \dots = y_1^n + \dots \qquad (n=1, 2)$

1. The magic square given on the right was the first which became known in the occident and is found in a copper-plate engraving by the German artist Albrecht Dürer.

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

Besides the usual magical relations, the square is interesting as it furnishes solutions of the simultaneous equations

$$(x_1,\ x_2,\ x_3,\ x_4)^n = (y_1,\ y_2,\ y_3,\ y_4)^n \qquad n=1,\ 2 \qquad \dots \quad (1)^*$$

Thus we have for $n=1, 2$

$$(16,\ 3,\ 2,\ 13)^n = (4,\ 15,\ 14,\ 1)^n;\ (5,\ 10,\ 11,\ 8)^n = (9,\ 6,\ 7,\ 12)^n$$

$$(16,\ 5,\ 9,\ 4)^n = (13,\ 8,\ 12,\ 1)^n;\ (3,\ 10,\ 6,\ 15)^n = (2,\ 11,\ 7,\ 12)^n$$

We also notice the relations

$$x_1 + y_4 = x_2 + y_3 = x_3 + y_2 = x_4 + y_1 \qquad \dots \quad (2)$$

From (1) and (2), it follows that

$$x_1x_2 + x_2x_3 + x_3x_4 + x_4x_1 = y_1y_2 + y_2y_3 + y_3y_4 + y_4y_1 \qquad \dots \quad (3)$$

as may be easily verified.

2. From any given solutions of (1) we may get others, for we may multiply all the elements by the same factor or add the same constant to all the elements without affecting (1). It is also easy to see that from (1), it follows that

$$(x_1 + cy_1,\ \dots x_4 + cy_4)^n = (cx_1 + y_1,\ \dots cx_4 + y_4)^n \qquad (n=1,\ 2)$$

where c is any number.

3. If, however, x_r and y_r satisfy the relations (1) and (2) and are both in ascending order of magnitude, we have also

$$x_1x_4 + x_2x_3 = y_1y_4 + y_2y_3$$

Thus $2 \times 16 + 3 \times 13 = 1 \times 15 + 4 \times 14$. It follows that

$$(x_1 + ax_4,\ x_2 + ax_3,\ \dots\ x_4 + ax_1)^n = (y_1 + ay_4, \dots y_4 + ay_1)^n \qquad (n=1,\ 2)$$

Thus from $(2, 3, 13, 16)^n = (1, 4, 14, 15)^n$ $(n = 1, 2)$ we have, using the above result with $a=3$, $(50, 42, 22, 22)^n = (46, 46, 26, 18)^n (n=1, 2)$. Dividing each by 2^n and subtracting 11, we have

$(14, 10)^n = (12, 12, 2, -2)^n$ $(n=1, 2)$ which gives a solution of

$$x_1^n + x_2^n = y_1^n + y_2^n + y_3^n + y_4^n \qquad (n=1,\ 2).$$

* $(x_1, x_2, x_3, x_4)^n$ stands for $x_1^n + x_2^n + x_3^n + x_4^n$.

4. From $(x_1, x_2 \ldots x_r)^n = (y_1, y_2 \ldots y_r)^n$ $(n=1, 2)$ we get

$$(x_1, x_2 \ldots y_1+t, y_2+t, \ldots)^n = (y_1, y_2 \ldots x_1+t, x_2+t, \ldots)^n \qquad (n=1, 2, 3)$$

where t is arbitrary, as may be easily verified. It is also worth noting that if a, b, c, d are four numbers in arithmetic progression, then

$$(x_1+a, \ldots x_4+d)^n = (y_1+a, \ldots y_4+d)^n \qquad (n=1, 2)$$

Thus from $(4, 5, 9, 16)^n = (1, 8, 12, 13)^n$ $(n=1, 2)$ and the progression 2, 5, 8, 11 we have also

$$(6, 10, 17, 27)^n = (3, 13, 20, 24)^n \ (n=1, 2).$$

Nurnberg
Germany

ALFRED MÖESSNER

An Intriguing Fallacy

This fallacy occurs in a piece of geometrical reasoning that came in my way in tackling a problem. This problem, the arguments on it and the conclusion reached are given below. On proceeding from the conclusion a little further, by way of drawing a corollary, a result is reached which is manifestly absurd. That shows there is a fallacy somewhere; but WHERE is the question. I begin with the statement of two theorems for ready reference, or as introduction.

Two Theorems

THEOREM I.—Given a triangle ABC and a point M, the locus of a point P whose pedal circle w. r. t. triangle ABC passes through M is a cubic curve. It circumscribes the triangle ABC; and we shall denote it by π (ABC)M.

THEOREM II.—If we have a quadrangle ABCD and a point P, then, the pedal circles of P w. r. t. triangles DBC, DCA, DAB and ABC concur at a point. We shall denote the point by M.

Here, let it be observed that if we take a pair of these pedal circles, for example, w. r. t. triangles DBC and ABC, one of their intersections is the foot of the perpendicular from P on BC. We shall call it their *primary* intersection. Then M will be their *secondary* intersection. It is this secondary intersection of each pair that is common to all the four pedal circles.

Problem

The problem in question is the converse of Theorem II. If we are given a quadrangle ABCD and a point M, it is required to find the point P, or the points P (and their number), whose pedal circles w. r. t. the triangles of the quadrangle ABCD concur at M.

Arguments

We proceed thus :—By theorem I, P lies on the cubic denoted by π (ABC)N. For brevity we shall omit the suffix M and call it π(ABC). Similarly, P lies on the cubic denoted by π(DBC). These two cubics intersect in 9 points ; of which two are B and C. There are 7 remaining points of intersection which we shall call *the set* β. Let P be any point of the set β. Then, the pedal circles of P. w. r. t. triangles ABC and DBC pass through M ; and M, obviously, is their *secondary* intersection, for M need not lie on BC. Therefore the pedal circles of P. w. r. t. triangle DCA and DAB, by theorem II, also pass through M.

That is, P also lies on the cubics denoted by π (DCA) and π(DAB). This holds for every point of the set β.

Conclusion

We conclude (1) that the cubics denoted by π(ABC), π(DBC), π(DCA) and π (DAB) have 7 common points forming the set β ; (2) that, taken two by two, these cubics intersect in two additional points forming a pair of vertices of the quadrangle ABCD ; and (3) that the seven points of the set β are the points whose pedal circles w. r. t. triangles of the quadrangle ABCD pass through M.

A Revealing Corollary

Let us here take note of a well-known theorem :

THEOREM III.—If we have 9 points forming the complete intersections of two cubics, any cubic that passes through 8 of them passes also through the ninth.

Consider now the three pairs of cubics :

π(DCA), π(DAB) ; π(DAB), π(DBC) ; π(DBC), π(DCA).

The complete intersections of the first pair are the seven points of set β together with D and A. Hence, we infer that all cubics passing through the seven points of set β *and* D must pass through A. From like consideration of the second and third pairs we infer that the cubics passing through the seven points of set β and D must pass also through B, and through C. That is, we have a pencil of cubics having 11 points in common !

This is absurd. We conclude there is a fallacy in the reasoning somewhere. It is necessary to 'fix' it and find the true state of the geometrical facts connected with the problem.

I was able to resolve the puzzle about a week after I got into it. The number of points P is just 3 and not 7. The fallacy is somewhat deep-seated and I hope there will be readers contributing to clear it up.

V. RAMASWAMI AIYAR.

SOLUTIONS TO QUESTIONS

Question 1624

(K. J. SANJANA, M. A.). Give a strict demonstration of the following theorem, which is the converse of a well-known theorem of Fermat.

The squares of two numbers, which are prime to each other are added together; prove that the sum is, except in one obvious case, a prime number of the form $(4M+1)$ or a product of prime numbers of this form. The exception occurs when the numbers squared are both odd, in which case the sum contains the factor 2, which though a prime number is not of the form $4M+1$. When this factor is removed, the remaining number has the property enunciated above.

Solution by K. B. Krishna Rao.

We will first show that if m and n be prime to each other, every odd prime divisor of m^2+n^2 is of the form $4M+1$. Let $2k+1$ be an odd prime divisor of m^2+n^2. Then m and n are each prime to $2k+1$, being prime to one another.

$\therefore\ m^{2k}-1 \equiv 0 \pmod{2k+1} = M(2k+1)$. Also $m^2+n^2 = M(2k+1)$.

$\therefore\ \{M(2k+1)-n^2\}^k = m^{2k} = 1+M(2k+1)$.

$\therefore\ (-1)^k n^{2k} = 1+M(2k+1)$.

But $n^{2k} = 1+M\ (2k+1)$, n and $2k+1$ being prime to one another. Hence $(-1)^k$ is positive (i.e.) k is even.

Thus every odd prime divisor of m^2+n^2 is of the form $4M+1$.

Case (i) :—If m and n be not both odd, m^2+n^2 is odd. m and n cannot be both even being prime to one another. Hence the prime 2 is not a factor of m^2+n^2 and m^2+n^2 is of the form $4M+1$ being composed of odd prime factors of the form $4M+1$.

Case (ii) :—When both m and n are odd we have $m^2+n^2=2N$, where N is odd. Now every old prime divisor of m^2+n^2, being prime to 2, is a factor of N. Thus all odd prime divisors of m^2+n^2 are factors of N. Clearly N is composed of only the odd prime divisors of m^2+n^2, and they being each of the form $4M+1$, N is also of the same form $4M+1$.

Question 1662

(T. R. RAGHAVASASTRI) :—A, B, C and P are four fixed points in a plane. Show that the locus of the Frégier points of P for all conics passing through ABCP is a straight line. Show that this straight line

(i) passes through a fixed point as P traces the circumcircle of ABC;

(ii) envelops a parabola inscribed in the triangle ABC as P moves on a straight line passing through the orthocentre of the triangle ABC.

(1) *Solution by A. Ranganatha Rao.*

Reciprocating with respect to P, the conics through ABCP transform into parabolas touching three lines $a\ b\ c$ and the Frégier points into the directrices of the parabolas. As the directrices are concurrent at the orthocentre of abc, the locus of the Frégier points is a straight line L.

(i) Since the Frégier point for a circle is always its centre, as P moves on the circumcircle of ABC, L always contains the circumcentre of ABC.

(ii) Considering the three line-pairs of the system of conics through ABCP, we see that L contains the points of intersection of BC, CA, AB with the perpendiculars at P to AP, BP, CP respectively. Thus the three diagonals of the complete quadrilateral formed by the lines BC,CA,AB,L subtend an orthogonal involution pencil at P. Hence the pair of tangents from P to any conic touching these four lines are at right angles. In particular, the directrix of the parabola touching these four lines passes through P (and also through the orthocentre H of ABC.) Thus, as P moves on fixed line through H, L touches the fixed parabola inscribed in ABC, which has PH for directrix.

Also solved by the same method by G. V. Ramachandran.

(2) *Solution by Nathan Altshiller-Court.*

Let a, b, c, be the lines perpendicular at P to the lines PA, PB, PC, and let K, L, be the points of intersection of the lines a, b, with any conic Σ of the pencil determined by the four given points A, B, C, P. From the hexagon AKCBLP inscribed in Σ we conclude, by Pascal's theorem, that the three points G = (AK, BL), M = (KC, LP), N = (BC, PA) are collinear. Thus the Frégier point G of P for Σ is

the intersection of the lines AK and NM. When Σ varies describing the pencil ABCP, the point N remains fixed, and the point K varies on the fixed line a, hence

A(K...) $\sim$ (K...) $\sim$ C(K...) $\sim$ (M...) $\sim$ N(M...).* The centers of the two pencils A(K...), N(M...) being collinear with the point P common to the bases a, b, of the two ranges (K...), (M...), the locus of the Frégier point G = (AK, NM) is a straight line f.

When K coincides with the point D = (a, NBC) the above construction shows that D belongs to the locus f of G. Now in the construction of the locus f we made use of the lines a, b But any two of the three lines a, b, c, could have served this purpose, hence f also passes through the points E = (b, CA), F = (c, AB). The line f = DEF may thus be constructed without the help of any conic of the pencil ABCP. It may also be pointed out that this proof provides its own check. Indeed, the three pairs of lines a, PA; b, PB; c, PC, form an orthogonal involution having P for center, hence the lines a, b, c, meet the respective sides of ABC in the three collinear points (M. Chasles, *Géométrie supérieure*, p. 247).

(i) The point G is the pole of the involution of points which the fixed involution of rays a, PA; b, PB; c, PC, determines on the variable conic $\sum$ of the pencil ABCP, and the line f is the locus of this pole. If the four points A, B, C, P lie on a circle (O), this circle is a conic of the pencil, and the involution determined on (O) by the three pairs of rectangular lines obviously has for its pole the center O of (O), hence the line f passes through O. If P varies on the circumcircle of ABC, the line f turns about the center O of this circle.

(ii) Let P be any point on the given line s, and let D be the trace on BC of the perpendicular to PA at P. The circle (O_d) having AD for diameter passes through P and through the foot U of the altitude AU of ABC. Let P′ be the second point of intersection of this circle with the line s. Since AP′D is a right angle, to the point P′ of s corresponds on BC the same point D as to P. As the circle (O_d) varies describing the coaxal pencil having for basic points A and U, the range (D...) described by D on BC and the involution (PP′,...) described by P and P′ on s are both projective to the pencil of circles and therefore to each other.

Let E be the trace on CA of the perpendicular PE to BP at P. The circle (O_e) described on BE as diameter passes through the foot

* $\sim$ means "is projective with."

V of the altitude BV of ABC, and if P″ is the second point of intersection of (O_e) with s, we shall have, as above,

$$(E...) \sim (PP'',....)$$

Since the line s passes through the orthocenter H of ABC, we have from the two circles (O_d) and (O_e),

$$HP \cdot HP' = HA \cdot HU, \qquad HP \cdot HP'' = HB \cdot HV$$

But the second terms of these two equalities are equal, hence the same is true about the first terms, i.e., the points P′ and P″ coincide, and the two involutions considered are identical, H being their central point. Thus the two ranges (D...), (E...) are projective to the same involution and therefore projective to each other, hence the line DE envelopes a conic tangent to BC and CA. To the point H of s correspond on BC and CA the points at infinity on these two lines, hence the line at infinity is tangent to the conic which is therefore a parabola (P). This curve is also tangent to AB, for in the determination of (P) we could have made use of any two of the three sides of the triangle ABC.

(3) *Remarks by B. Ramamurti.*

As is evident from the first solution, the locus of the Frégier points of P for all conics through ABCP is the ortho-line p of P with respect to the triangle ABC (i.e.) the line of collinearity of the points of intersection with BC, CA, AB, of the lines through P perpendicular to PA, PB, PC respectively. The transformation P-p is studied, by K. Satyanarayana * and K. Rangaswamy. The ortho-line p may also be regarded as the directrix of a conic, having P as the corresponding focus and ABC as a self-conjugate triangle. Given p, there are two positions P_1, P_2 for P and P_1, P_2 are inverse points with respect to the polar circle of triangle ABC.

Let us briefly consider some results which are noticeable in the light of the above ideas. If P_1 moves on the circumcircle of ABC, P_2 moves on the nine-point circle of ABC. Hence we have in addition to (1).

The ortho-lines with respect to a triangle of points or the nine-point circle are concurrent at the circumcentre. (1′)

* Satyanarayana "On conics having a common self-conjugate triangle". *Mathematics Student* I. 19-24. Rangaswamy: On the ortho-point-line transformation, Vol. 2, pp. 151-153.

From (1) and (1′) and the interpretation above we have

If a focus of a conic lies either on the circumcircle, or on the nine-point circle of a self-conjugate triangle, the corresponding directrix passes through the circumcentre.

Let us now generalise result (2). If P_1 moves on a line through the orthocentre O, P_2 also moves on the same line, since the line is its own inverse with respect to the polar circle. If instead of a degenerate orthogonal circle, viz. a line through O, we take any proper circle orthogonal to the polar circle, we have

If a point P moves on a circle S orthogonal to the polar circle of a triangle, the envelope of the ortho-line of P is the conic touching the sides of the triangle, and having S as its director circle. (2)

To prove this we take any point D on BC and the points P or S whose ortho-lines pass through D are the points of intersection with S of the circle on AD as diameter. Since the two circles are both orthogonal to the polar circle, their points of intersection P_1, P_2 are inverses with respect to the polar circle and hence have the same ortho-line through D. Hence, as in the second solution, the relation between D and E is homographic, and the envelope of the ortho-line DEF is a conic touching BC, CA, and similarly AB also. Further since the circles on AD, BE, CF all pass through P_1, P_2, any pair P_1, P_2, lie on the director circle of the envelope of p. Hence S is the director circle of the envelope.

Question 1672

(V. RAMASWAMI AIYER.):—ABCD is a quadrilateral inscribed in a circle, centre O. BC, AD intersect at P. If X, Y are the circumcentres of the triangles ABP, CDP, show that the circumradii of these triangles are equal to OY, OX.

Solution by A. Ranganatha Rao.

Since triangles ABP, CDP are similar and X, Y are corresponding points connected with them, we have $\angle XPB = \angle YPD$. Hence PX, PY are isogonal conjugates with respect to the triangle APB. But, as X is the circumcentre of ABP, PY is perpendicular to AB. Similarly, PX is perpendicular to CD. Also XO and YO are the perpendicular bisectors of AB and CD respectively. Hence OYPX is a parallelogram. Therefore, $OX = YP$ and $OY = XP$.

Also solved by K. Rangaswami, S. P. Ranganathachar, M. V. Jambunathan, T. K. Raghavachari, N. K. Narasimhamurti, N. Maitra, Hansraj Gupta, V. Ranganathan, M. K. Srinivasa Rao, A. T. Doraiswamy Iyengar.

REVIEWS

G. S. MAHAJANI, M.A., Ph.D. *Lessons in Elementary Analysis.* Second Edition. Aryabhushan Press, Poona.

We are glad to welcome the second edition of Dr. Mahajani's Lessons in Elementary Analysis. The first edition which was published some five years ago differed from the usual text-books on the Calculus by its more careful and detailed treatment of the fundamental concepts of Modern Analysis—limits, continuity of functions of one variable and two variables, uniform convergence, inversion of operations etc. The fact that a second edition has been called for shews that the book has been generally adopted as a text-book for the graduate and M. A. classes, and that colleges are appreciating at its true worth the aspect of rigour and logical discipline in modern mathematics.

In the new edition, there has been some rearrangement of material and rewriting of some of the articles, suggested by the author's teaching experience. An introductory chapter on the Arithmetic continuum has been added. Among the other additions are Abel's Theorem on Power Series, Taylor's theorem for functions of several variables etc.

A few inaccuracies on pages 119, 121 and 122 have been subsequently corrected by the author in a separately printed sheet.

SRI NIWAS ASTHANA. *The Trisection of an angle;* 15 pp. 1934. Price Re. 1-8-0.

Mr. Sri Niwas Asthana is an assistant teacher in Shivpuri, Gwalior, who though not a mathematician "in the literal sense of the word", has felt the lure of the insoluble problems which are "still tormenting the mathematicians' brains" and has come out with his "researched mathematical truths". The booklet considers only the multisection of the angle, but the author promises to attempt the squaring of the circle in the next edition, in accordance with the advice—if we are to believe the introduction—of Prof. S. C. De.

The Author takes two concentric circles whose radii are as $1 : \sqrt{2}$, and notes that the chord of half the arc of the first is also the chord of $\frac{1}{3}$ of the arc of the second. On the implicit assumption that a

similar property holds for corresponding arcs subtending *any* angle at the centre, the author devises a solution of the problem.

The price of the booklet is evidently related more to the antiquity of the problem than to the cost of printing. Who would grudge to pay Re. 1-8-0 for a 15-page booklet with a paper cover solving an age-long problem —if it were correct.

A. N. R.

PHILIP KEMP : *Tables for the Analysis of Periodic Wave Forms.*

(Extracted from the Author's *Alternating Current Electrical Engineering.*)

These are two tables for the analysis of complex periodic wave forms occurring in Alternating current theory. In the ordinary type of A. C. generators having a constant speed the two halves of the wave are similar since whatever occurs under one pole is repeated under the next which is of opposite polarity. Hence only odd harmonics occur and the waves may be represented by

$$V_\theta = E_1 \sin(\theta + \alpha_1) + E_3 \sin(3\theta + \alpha_3) + \dots E_n \sin(n\theta + \alpha_n) \quad \dots(1)$$

where n is an odd integer.

In solving up to say the 17th harmonic ($n=17$), the range is divided into intervals of 10° and the mid-interval readings

V_5, V_{15} ... V_{175} are measured. Now from (1) it follows that

$$V_\theta + V_{180-\theta} = 2E_1 \cos\alpha_1 \sin\theta + 2E_3 \cos\alpha_3 \sin 3\theta \dots$$
$$\dots + 2E_{17} \cos\alpha_{17} \sin 17\theta \quad \dots \quad (2)$$
$$V_\theta - V_{180-\theta} = 2E_1 \sin\alpha_1 \cos\theta + 2E_3 \sin\alpha_3 \cos 3\,\theta + \dots$$
$$\dots + 2E_{17} \sin\alpha_{17} \cos 17\,\theta \quad \dots \quad (3)$$

Giving θ the values 5°, 15°,...85°, we have 9 linear equations (2) for solving the 9 unknowns $E_r \cos\alpha_r$ as linear functions of $V_5 + V_{175}$, $V_{15} + V_{165}$ etc., the multipliers occurring therein being absolute constants (1/9 of sines of certain multiples of 5°) which may be calculated once for all. In the same manner, the equations (3) give 9 linear equations for determining $E_r \sin\alpha_r$. The amplitudes E_r and the phases α_r of the several harmonics are thereby determined.

The sheet containing two tables corresponding to $n=17$ and $n=5$ is now extracted from the book and is separately available. The calculations can be carried out speedily by inserting the data and following the instructors in the sheet where the numeral multipliers are printed in the columns where they are required. A useful sheet for the practical Electrician and Physist which could be used without an acquaintance with the mathematical theory.

A. N. R.

G. F. C. SEARLE, Sc. D., F.R.S. *Experimental Physics, a selection of Experiments.* Cambridge University Press, 1934, pp. 363, Price 16/-.

This book which is written on the same lines as the author's well-known manuals on *Experimental Optics, Experimental Elasticity,* and *Experimental Harmonic Motion,* but covers a wider range will be warmly welcomed by all those working in Physical and Engineering laboratories. The mathematical theory of the relevant field is first expounded, the consequences as applied to a particular experiment worked out and the experimental technique elaborated in sufficient detail to enable the apparatus to be constructed and used, a further valuable aid being the dimensions and measurements obtained in an actual experiment on the lines suggested. The inevitable gap between the mathematical theory and the actual experimental work is sought to be bridged by a theoretical discussion of the optimum conditions for large observable effects and the corrections to be applied for factors ignored in the theoretical treatment and for the small discrepancies which it may not be possible to eliminate where the mathematical theory demands a coincidence.

The first chapter deals with experiments in rigid dynamics of which there are four, an ingenious one to illustrate the conservation of angular momentum, Kater's pendulum, one for the moment of inertia by a comparison of the times of oscillation of a compound pendulum with adjustable parts, and a recording gyroscope. The second chapter explains the important theory of the determination of the rotation speeds by means of patterns on the wheels examined by intermittent light and its application to determine the frequency of an alternating current. There are two chapters devoted to elasticity the second of which uses optical methods for determining elastic constants. Surface, Tension, Heat, and Sound claim two chapters each, the first of which is devoted to an exposition of the mathematical theory and the second to the experimental details, and one chapter dealing with experiments in viscosity.

The reviewer feels that in Indian Colleges mechanics tends to degenerate into a source of questions in differential equations, and would become more real if a short practical course could be gone through. The publication of the book should prove very helpful both for the experiments which it contains and those which it will suggest to the intelligent reader.

A. N. R.

BOOKS RECEIVED FOR REVIEW

ISAAC MATHAI, M.A. HONS.: *Matriculation Mathematical Tables.* 74 pp. Re. 1. Mathai's Publications, Bombay.

ANNOUNCEMENTS AND NEWS

Readers are invited to contribute to the value of this section by sending suitable news items of interest.

The following gentlemen have been elected members of the Society:

(1) D. C. Pavate Esq., M.A. (Cantab), Assistant Director of Public Instruction, Poona.

(2) A. S. Krishnamurthi Esq., P. W. D. Executive Engineer's Office, Chanda.

(3) R. D. Misra Esq., M.Sc., Lecturer, Lucknow University.

(4) M. Zia-ud-din, M.A., Professor, S. E. College, Bhawalpur.

The Committee of the Indian Mathematical Society has accepted the invitation of the Delhi University to hold its ninth Conference at Delhi between the 19th and 21st December 1935. A sum of Rs. 100 has been sanctioned out of the Society's funds. A local Committee has been appointed to make the necessary arrangements consisting of Dr. Ram Behari, M.A., Ph.D., (Chairman), Dr. Kothari, M.A., Ph.D. and Dr. Bhoj Raj Seth, M.A., Ph.D. All authors who wish to contribute papers for the conference are requested to send them with short abstracts to Mr. A. Narasinga Rao, Annamalai University, Annamalainagar so as to reach him before October.

We offer our hearty congratulations to Dr. T. Vijayaraghavan, B.Sc., D.Phil, Reader in Mathematics in the Dacca University and a member of the Society, on his appointment as visiting lecturer of the American Mathematical Society for 1936. Dr. Vijayaraghavan is the first Indian mathematician to receive this honour.

The Franklin medal was awarded to Albert Einstein and Sir Ambrose Fleming for their work in the fields of Relativity and Wireless Research respectively. Each Scientist will also get a 1000 dollars from the Franklin Medal Fund created by Mr. Samuel Insull.

The Annamalai University has decided to confer the M. Sc. research degree on Mr. K. Rangaswami, B. Sc. Honours, Research Student in mathematics, on the basis of his thesis on "The Theory of Normals to a Quadric in Hyperspace."

A new type of computing machine called the Trianalyst has been invented consisting of a rigid right triangle with graduated sliding scales in each of its legs; a movable hypotenuse pivoted at one end of an indicator which slides along one of the legs of the triangle; and a square with its edges perpendicular to the legs of the triangle and with its vertex sliding along the bisector of the right angle of the triangle. The instrument may be used for addition, subtraction, multiplication, division, and solution of equations of degrees up to the fourth and certain simultaneous equations in two unknowns.

The Proceedings of the National Institute of Sciences of India are now being received in exchange for the publications of the Society.

Daniel Alexander Murray, author of the well-known *Introductory Course of Differential Equations* and of several other text-books, died in October 1934.

The National Institute of Sciences for India.

The National Institute of Sciences for India was inaugurated by His Excellency Sir John Anderson, Governor of Bengal, at a meeting held on the 7th January 1935 at Calcutta, under the chairmanship of Dr. J. H. Hutton, President, Indian Science Congress. Thereafter Dr. L. L. Fermor, the President of the National Institute, delivered the inaugural address on "The Organisation of Scientific Research in India."

The principal function of the Institute will be to co-ordinate the labours of all scientists in India, to effect co-operation between the various bodies of Academy rank and to render possible the formation of a National Research Council to co-operate with the International Research Council. In order to provide for *liaison* with the three bodies of Academy rank (the Asiatic Society of Bengal, the U. P. Academy of Sciences, and the Indian Academy of Sciences, Bangalore)

there is provision on the Council of the Institute for an additional Vice-President and additional member of council nominated by each body, while there is a similar privilege also for the Indian Science Congress Association. There are to be 125 Foundation Fellows and 10 Fellows are to be elected annually. The Council of the Institute consists of 25 Fellows.

The National Institute proposes to issue four publications:

1. *Transactions* to be issued at irregular intervals;

2. *Proceedings*, recording the business of the institute and also containing short articles.

3. An Annotated *Bibliography of Science* in India containing abstracts of all papers read before the Institute or Academies or published in India.

All authors submitting papers for publication in the Journal of the Indian Mathematical Society or in the Mathematics Student are requested to send at the same time brief abstracts of their communications for publication in the Bibliography of Science in India published by the National Institute. These Abstracts may be sent to the Editor of the Student.

The office-bearers for 1935 are:

President: Dr. L. L. Fermor, Calcutta.

Vice-Presidents: Brigadier Dr. H. J. Couchman, Calcutta and Dr. B. Sahni, Lucknow.

Treasurer: Dr. S. L. Hora, Calcutta.

Secretaries: Dr. M. N. Saha, Allahabad; Dr. S. P. Agharkar, and Dr. A. M. Heron, Calcutta.

Headquarters: Calcutta.

QUESTIONS FOR SOLUTION

1700. (V. THEBAULT, Le Mans, France) with the figures 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 taken each once, form a perfect square whose root is an integer of the form $a\,b\,b\,b\,a$.

1701. (V. THEBAULT)—Chords are drawn to a sphere which are parallel and bear a constant ratio to chords of a second sphere which pass through a fixed point.

Show that the envelope of these chords is a surface of the second degree.

1702. (K. SATYANARAYANA)—The common points of the circles on the diagonals of a quadrilateral as diameters are such that each side is the ortho-line of either of the points with respect to the triangle formed by the other three. Calling these *the ortho-points of the quadrilateral*, prove that:

(1) the circum-circle of the diagonal triangle and the self-polar circles of the four triangles of the quadrilateral form a coaxal system with the ortho-points as limiting points.

(2) the director circles of all in-conics of the quadrilateral form a coaxal system with the ortho-points as common points.

Hence prove Gaskin's theorem.

Show also that the necessary and sufficient condition for the existence of a rectangular hyperbola in the in-conic family is the coincidence of the ortho-points.

1703. (V. RAMASWAMI AIYER, M.A.)—Prove that the Nine-Points circle of the diagonal triangle of any quadrilateral passes through the Miquel point of the quadrilateral.

1704. (A. NARASINGA RAO):—Show that for any tangential pencil of quadrics:

(1) there are, in general, three quadrics through a given point,

(2) the tangent planes to the 3 quadrics through a point and the 4 faces of the common self polar tetrahedron are osculating planes of the same twisted cubic.

1705. Show that the star (α, δ) will have a heliacal rising when the right ascension of the Sun is R, where

$$\tan R = \frac{\tan\delta\cos\alpha - \sin\alpha\sqrt{\cot^2\phi - \tan^2\delta}}{\tan\omega - \tan\delta\sin\alpha - \cos\alpha\sqrt{\cot^2\phi - \tan^2\delta}}$$

Hence deduce that:

(1) at the equator, the heliacal rising of a star occurs when $R = \alpha$;

(2) at the poles, when the star is on the celestial equator and the sun on the horizon, the star has a heliacal rising;

(3) an equitorial star will have a heliacal rising when R is given by

$$\tan R = \frac{\sin\alpha\cot\phi}{\cos\alpha\cot - \tan\omega}.$$

(4) The first point of Aries has a heliacal rising, if the colatitude of the place is equal to the obliquity of the ecliptic. Then, the first point of Aries rises with the Sun daily during a half year and the first point of Libra during the other half.

1706. (M. R. P. AYYANGAR). Show that for a frictionless fluid

$$\int\frac{dp}{\rho} + v + \tfrac{1}{2}q^2 + \sum\omega_1(zv - yw) + \tfrac{1}{2}\sum y^2(\omega_3 + \omega_1)\omega_2$$

$$- \sum\omega_2^2(x + z) - \tfrac{1}{2}\sum\frac{d\omega_3}{dt}(y^2 - x^2) - \frac{\partial\varphi}{\partial t} = F(t).$$

where u, v, w are the components of the velocity at time t at the point x, y, z relative to moving axes having component angular velocities $w_1\ w_2\ w_3$.

190018

ATHEMATICAL ASSOCIATION OF AMERICA

.l Association of America is devoted primarily to the interests ıtics. It now has over two thousand individual and institu- ıere are twenty sections at present, representing thirty-one he Association holds two national meetings per year and the ɔr two meetings per year. All meetings, both national and sectional, are reported in the official Journal.

Application blanks for membership may be obtained from the Secretary, Prof. W. D. Cairns, Oberlin, Ohio.

THE AMERICAN MATHEMATICAL MONTHLY

Official Organ of

THE MATHEMATICAL ASSOCIATION OF AMERICA

Is the Only Journal of Collegiate Grade in the Mathematical Field in this Country.

Most of its mathematical contributions can be read and understood by those who have not specialized in mathematics beyond the Calculus.

The historical papers, which are numerous and of high grade, are based upon original research.

The Questions and discussions, which are timely and interesting, cover a wide variety of topics.

Surveys of the contents of recent books constitute a valuable guide to current mathematical literature.

The topics in the department of Undergraduate Mathematics Clubs have excited wide interest among those engaged in teaching mathematics.

The news and notices cover a wide range of interest and information both in this country and in foreign countries.

The Problems and Solutions hold the attention and activity of a large number of persons who are lovers of mathematics for its own sake.

"CURRENT SCIENCE"

A MONTHLY JOURNAL OF SCIENCE

Contains readable and authoritative articles; and short communications regarding new achievements in all branches of Science.

Annual Subscription Rs. 6/– only.

For further particulars apply to

THE SECRETARY,
"CURRENT SCIENCE,"
Indian Institute of Science,
HEBBAL P. O., Bangalore.

VOLUME III, NUMBER

MARCH, 1935

CONTENTS

THE NUMBER CONCEPT: By **Rev. J. B. Freeman** 1

THE BACKWARDNESS OF EARLY AMERICAN MATHEMATICS: By **G. A. Miller** ... 8

THE CORRECTIONS OF 'VARIATION' AND 'EVECTION' IN HINDU ASTRONOMY: By **D. Akku Somayajulu** 12

ON THE NATURE OF THE ROOTS OF CERTAIN EQUATIONS: By **M. N. Narasimha Iyengar** 19

NOTES AND DISCUSSIONS

On Sums of like Powers: By **K. Subba Rao** 23

On the Equation: By **Alfred Möessener** 24

An Intriguing Fallacy: By **V. Ramaswami Aiyar** 25

SOLUTIONS TO QUESTIONS 27

REVIEWS 32

BOOKS RECEIVED FOR REVIEW 35

ANNOUNCEMENTS AND NEWS 35

QUESTIONS FOR SOLUTION 38

GLEANINGS 11

All communications intended for publication should be sent to the Editor, A. NARASINGA RAO, Annamalai University, Annamalainagar, South India. Communications relating to the receipt of the Journal, changes of address, and the purchase of current or back numbers of this journal should be sent to the Assistant Secretary, S. MAHADEVA IYER, Assistant Professor of Mathematics, Presidency College, Madras.

Remittances of membership fees and of subscriptions should be sent to the Treasurer, Prof. L. N. SUBRAMANIAM, Department of Mathematics, Christian College, Madras.